Driven Pair Contact Process with Diffusion

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The pair contact process with diffusion (PCPD) has been recently investigated extensively, but its critical behavior is not yet clearly established. By introducing biased diffusion, we show that the external driving is relevant and the driven PCPD exhibits a mean-field-type critical behavior even in one dimension. In systems which can be described by a single-species bosonic field theory, the Galilean invariance guarantees that the driving is irrelevant. The well-established directed percolation (DP) and parity conserving (PC) classes are such examples. This leads us to conclude that the PCPD universality class should be distinct from the DP or PC class. Moreover, it implies that the PCPD is generically a multi-species model and a field theory of two species is suitable for proper description.

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The steady state of an equilibrium system is characterized by its Hamiltonian and Gibbs measure. There is no systematic generalization to the stationary state of nonequilibrium systems so far. Since nonequilibrium systems encompass all kinds of many body systems without a constraint of detailed balance, it may be hopeless to find a universal formalism applied to general nonequilibrium systems. At this point, model studies or case-by-case studies are rather useful to accumulate our knowledge on nonequilibrium systems.

Our experience on equilibrium systems illustrates the scale-free fluctuation or power law behavior at the critical point where the continuous phase change occurs. The scale-free nature is worth while to be studied not only because of its theoretical attraction, but also because of ubiquity in nature – the clustering of galaxies [1], $1/f$ noise [2], percolation structure [3], to name only a few. This scale-free nature is also expected at criticality under nonequilibrium circumstances. As a prototype of nonequilibrium critical phenomena, absorbing phase transitions (APTs) have been studied extensively [4]. APT is a transition from an active phase to an absorbing phase in nonequilibrium steady states. The absorbing states are defined as the configurations where the system cannot escape by the prescribed dynamic rules. As in equilibrium systems, this transition is possible only at the thermodynamic limit because the finite systems eventually fall into the absorbing states.

In epidemiology, for example, the virus extinct state is an absorbing state. Actually, the disease spreading is modeled and dubbed the contact process (CP) by Harris [5]. CP shows a phase transition from the virus infested state (active state), to the quiescent state (absorbing state). This transition is known to belong to the directed percolation (DP) universality class. Actually, many types of models belong to the DP class and it is conjectured that a phase transition occurred in a system with a single absorbing state should share the critical behavior with the DP [6, 7].

As in equilibrium critical phenomena, a symmetry or conservation may play an important role in determining the universality class. Accordingly, many nonequilibrium systems with symmetric absorbing states or conservation laws are studied. As expected, the additional symmetry or conservation brings forth a series of new universality classes. Unfortunately, the absorbing states with higher symmetry or complex conservation are usually unstable with respect to an infinitesimal activity even in one dimension. Therefore, it is difficult to find a nontrivial scaling other than the mean-field type, except for a few well-established universality classes like the DP and the directed Ising or the parity-conserving (PC) classes.

In this context, the critical behavior of the pair contact process with diffusion (PCPD) [8] is rather surprising. Although the PCPD has no symmetry in absorbing states and no conservation law, the PCPD seems to form a new universality class. Actually, some authors asserted that PCPD eventually flows into the DP fixed point after a huge crossover time [9]. However, the extensive numerical experiments [10] indicate that the PCPD belongs to a new universality class other than the DP or the PC. In addition, the long-term memory present in the PCPD has been suggested as a source for this new universality class [11]. Nevertheless, the universality issue on the PCPD is still in hot controversy and it is not yet clearly settled down [8]. There have been some analytic attempts to analyze the PCPD through a single-species bosonic field theory, but no satisfactory results have appeared as yet [12].

In this Letter, we introduce external driving (biased diffusion) in various models including the PCPD and numerically observe its effect on the critical scaling. The external driving may serve as a crucial test on the universality class of the general absorbing-type models and also reveal important features of their critical scaling. With this test, we show later that the PCPD class should be distinct from the DP or the PC class and the PCPD is generically a two-species reaction-diffusion model.

The role of driving is usually irrelevant in single-species reaction-diffusion systems with absorbing states (SRDA). The simplest examples are the pair annihilation/coagulation models represented by $2A \to 0$ or $2A \to A$. These models can be solved exactly even with biased diffusion which turns out to be irrelevant to the long time decay dynamics of the particle density [13].

In the field theoretical sense, it is easily predictable within the bosonic formalism introduced by Doi, Grassberger, and others [14]. Since the particle density is so low in the long time limit, it would not be harmful to adopt the bosonic formalism where multiple occupations are allowed at a site. After taking a suitable modification of the dynamic rules for bosonic particles and developing the coherent-state path integral from the master equation [15], one can obtain the proper action S which can be treated by the systematic many-body analysis like renormalization group (RG) calculation [16]. Including biased diffusion (drift), the action for the pair annihilation/coagulation model is given as

$$\mathcal{S} = \int dt d\boldsymbol{x} \left[\bar{\varphi}(\partial_t - D\nabla^2 + \boldsymbol{v} \cdot \nabla)\varphi + \lambda_1 \bar{\varphi}\varphi^2 + \lambda_2 \bar{\varphi}^2 \varphi^2 \right], \tag{1}$$

where D is the diffusion constant and $\boldsymbol{v}$ is the drift velocity, while λ_1 and λ_2 are properly scaled reaction parameters. The particle density field is denoted by φ and its response field by $\bar{\varphi}$. The driving term can be simply gauged away by a Galilean transformation such as $\varphi(t, \boldsymbol{x}) \to \varphi(t, \boldsymbol{x} - \boldsymbol{v}t)$ and $\bar{\varphi}(t, \boldsymbol{x}) \to \bar{\varphi}(t, \boldsymbol{x} - \boldsymbol{v}t)$. Therefore, one can conclude that the driving is irrelevant for the pair annihilation/coagulation model in the long time regime.

The argument based on the Galilean invariance can be applied to more general SRDA exhibiting absorbing phase transitions. Near the transition, the particle density is low enough to assure the validity of the bosonic field theory. Only exceptions are found in some multi-species diffusion-reaction systems, where the hard core exclusion becomes crucial [17, 18]. The DP class is well known to be described by a single-species bosonic field theory as well as the PC class. Therefore, one can expect that the external driving does not change the critical scaling.

To confirm our expectation, we study the driven branching annihilating random walks with one (DBAW1) and two (DBAW2) offspring in one dimension. The models without the external driving, the BAW1 and the BAW2, belong to the DP and the PC class, respectively. The evolution dynamics for the driven models with fully biased diffusion are summarized using stoichiometric notations as

$$\begin{aligned} &A\emptyset \xrightarrow{p} \emptyset A, \quad AA \xrightarrow{p} \emptyset\emptyset, \\ &\begin{cases} A\emptyset \xrightarrow{1-p} AA, & \text{for one offspring,} \\ A\emptyset\emptyset \xrightarrow{1-p} AAA. & \text{for two offsprings.} \end{cases} \end{aligned} \tag{2}$$

For simplicity, the branching process is also taken to be biased, but this choice does not change our conclusion.

We perform Monte Carlo simulations starting with the fully occupied initial condition. The particle density $\rho(t)$ is measured as a function of time t in a lattice of size $L = 2 \times 10^5$ and $L = 10^6$ for the DBAW1 and the DBAW2, respectively. Up to the observation time, all samples are alive in our simulations. Since a power law decay $\rho(t) \sim t^{-\delta}$ is expected at criticality, one should look for a flat line in the $\rho(t)t^{\delta}$ vs t plot to locate the critical point. In Fig. 1, we find $p_c = 0.18825(5)$ with $\delta = 0.159(1)$ for DBAW1 and $p_c = 0.5332(2)$ with $\delta = 0.285(1)$ for DBAW2. The values of the critical exponent ratios agree perfectly well with the known values for the DP and the PC class. The driven systems with partial bias also show the same critical behaviors. This is exactly what we expected from the Galilean invariance argument for the SRDA.

Now, we turn to the PCPD model and study the effect of driving on its critical scaling. The model dynamics consists of three configurational changes such as (biased) diffusion ($A\emptyset \leftrightarrow \emptyset A$), pair annihilation ($2A \to \emptyset$), and creation of a particle by a pair ($2A \to 3A$). The algorithm to simulate the driven PCPD (DPCPD) in one di-

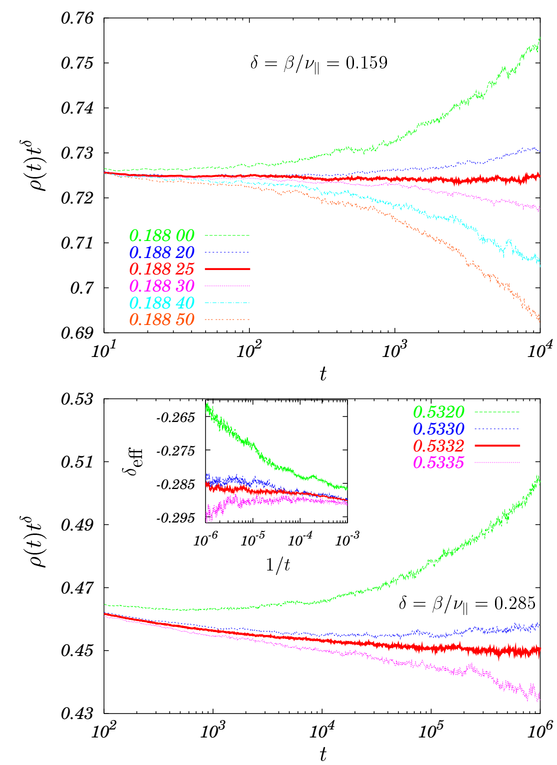

FIG. 1: (color online) Semi-log plots of $\rho(t)t^{\delta}$ vs t for DBAW1 (upper panel) and DBAW2 (lower panel). We use $\delta = 0.159$ for DBAW1 and 0.285 for DBAW2. Since DBAW2 shows a long-term correction to scaling, we also draw the effective exponents in the inset of the lower panel and find $\delta \simeq 0.285(1)$ for DBAW2 which is consistent with the PC value.

mension is as follows: First, choose a particle at random. The chosen particle attempts to hop to the right or to the left with probability D and $1-D$, respectively. If the target site is vacant, the hopping trial is accepted. If the target site is occupied, (a) two particles annihilate with probability p or (b) the hopping attempt is rejected and the pair (chosen particle and one at the target site) tries to create a particle at a randomly chosen nearest neighbor site of the pair. When the selected site is occupied, this branching attempt is rejected. The time increases by $1/N(t)$, where $N(t)$ is the total number of particles at time t.

We measure the particle density $\rho_s(t)$ and the nearest neighbor pair density $\rho_p(t)$ in a lattice of size $L = 10^7$ up to $t = 10^8$ and average over ~ 80 independent samples. At $D = 1/2$, the ordinary PCPD with normal diffusion is recovered. In Fig. 2, after a huge crossover time around $t \simeq 10^5$, we see a flat straight line at criticality ($p_c = 0.133522(2)$) with $\delta = 0.20(1)$ for both particle and pair densities, which is in good agreement with the most reliable value for the PCPD [10].

To see the effect of driving in the PCPD, we perform simulations at $D = 1$ (full bias). In Fig. 3, we find $p_c = 0.151031(1)$ with $\delta_s = 0.49(1)$ for the particle density and $\delta_p = 0.56(3)$ for the pair density, which are unambiguously distinct from the value of the ordinary PCPD, $\delta \simeq 0.20$. These results do not change for any partial bias. This is a big surprise because it implies that the ordinary Galilean invariance should not hold in the PCPD under driving, which in turn can not be described by a single-species bosonic field theory.

Before going into detailed discussion on its implication, we note that there is another surprise that the exponent values are almost identical to the values of the ordinary PCPD in two dimensions ("mean-field" values) [19]. The upper critical dimension of the PCPD is expected to be two and the decay dynamics presumably carries a multiplicative logarithmic factor in two dimensions. We plot $\rho_s(t)/\rho_p(t)$ versus t in a semi-log scale in Fig. 4, as in the 2D case studied in [19]. It seems to confirm that $\rho_s(t)/\rho_p(t) \sim \ln t$. Therefore, the critical scaling of ρ_s and ρ_p exhibits exactly the same critical behavior found at the 2D PCPD criticality. This may suggest that the upper critical dimension of the DPCPD is 1 rather than 2. The reduction of the upper critical dimension by the biased diffusion is not rare. The most prominent example is the sandpile model related to the self-organized criticality. It is well known that the directed toppling rules lower the upper critical dimension from 4 to 3 [20].

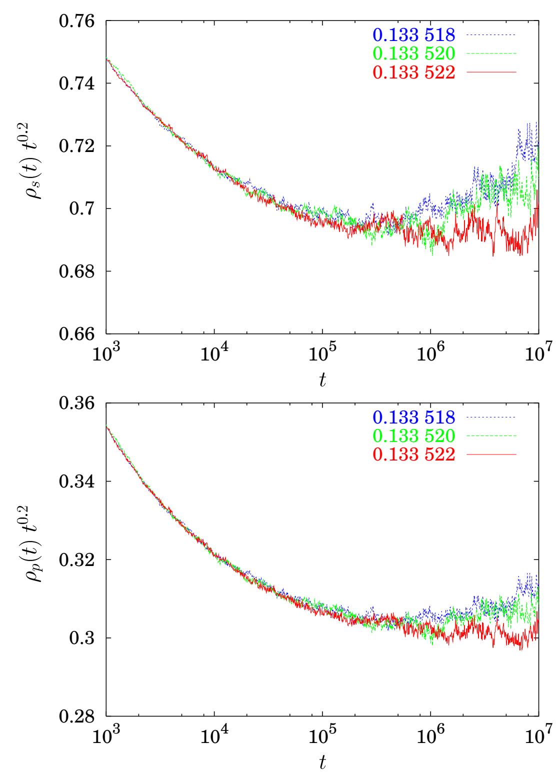

FIG. 2: (color online) A semi-log plot of $\rho(t)t^{\delta}$ vs t for the ordinary PCPD with normal diffusion ($D = 1/2$). In the upper (lower) panel, the data for the particle (pair) density are plotted. We find a good flat line at criticality with $\delta = 0.20$.

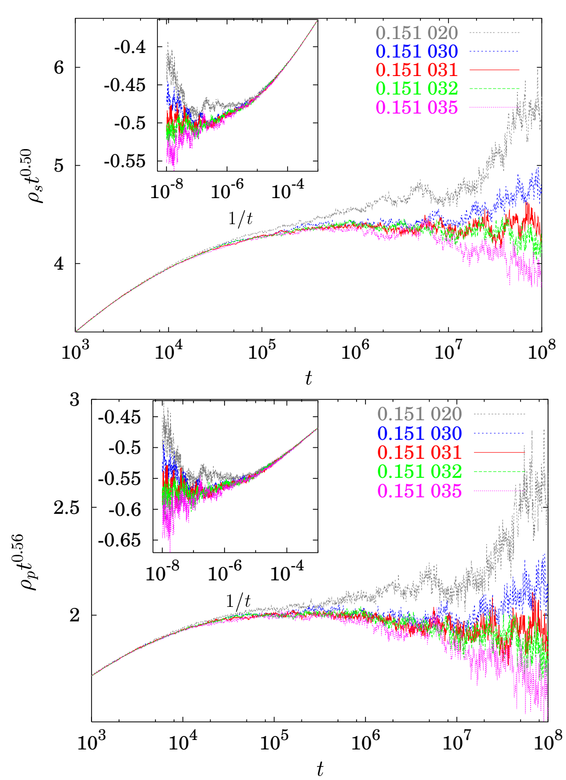

FIG. 3: (color online) Plots of $\rho_s(t)t^{0.49}$ vs t for the particle density and $\rho_p(t)t^{0.56}$ vs t for the pair density of the DPCPD model. In the inset of each panel, the effective exponents are drawn as a function of $1/t$.

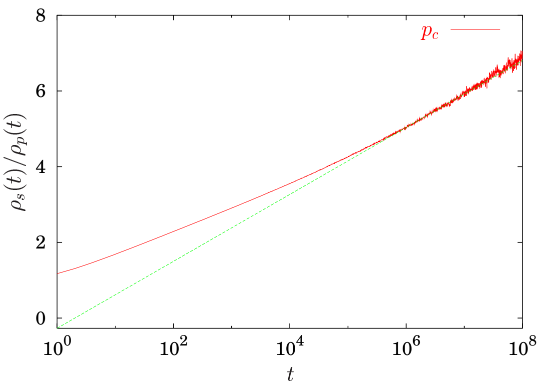

FIG. 4: (color online) The semi-log plot of ρ_s/ρ_p vs t at criticality. The straight line stands for the logarithmic fitting of the data, which seems very good for nearly three decades.

However, the situation is not so simple here. The decay dynamics inside the absorbing phase remains to be one dimensional, i.e. $\rho_s \sim t^{-1/2}$ and $\rho_p \sim t^{-3/2}$. Therefore, only the critical scaling carries the 2D character, while the absorbing phase is of 1D characteristic. The underlying mechanism for this surprising scaling behavior is under investigation.

Now, we come back to the implication given by the relevancy of the external driving. It implies that the PCPD under driving can not be described by a single-species bosonic field theory. This reminds us of the interpretation of the PCPD as a cyclically coupled DP and annihilation process suggested by Hinrichsen [21], where a pair and a solitary particle can be considered two independent excitations (two-species particles). If we accept that these two excitations are independent, a field theory of two-species is more suitable for the description of the PCPD. Then, the difference in the bias strength (drift velocity) for two different particles may be relevant as in the well-known two-species annihilation model $A + B \to \emptyset$ [22]. By introducing the biased diffusion of a single particle in the PCPD, the effective diffusion of a pair will be also biased but the drift velocity should be in general different each other. Therefore, our results suggest that the bias difference between two excitations are the reason for the relevancy of the driving in the PCPD in the context of a two-species reaction diffusion model.

In order to understand this feature more clearly, we study the full bosonic model with a *soft* constraint introduced by Kockelkoren and Chatè [10], which belongs to the PCPD class. It is obvious that the biased diffusion does not change the critical scaling in this full bosonic model due to the Galilean invariance. However, this is very fortuitous. Once we apply the different diffusion bias to a particle at singly occupied sites and a particle at multiply occupied sites, we recover the mean-field exponents again [23]. This confirms the role of the bias difference as a relevant perturbation to the PCPD fixed point.

We emphasize that the the bias difference is irrelevant for the multi-species models belonging to the DP class, because the DP is generically a single-species model. To check it explicitly, we study the generalized PCPD (GPCPD) model introduced by Noh and Park [11], which is parametrized by the memory strength r. At $r = 1$, the PCPD model is recovered, while the DP class is found at $r = 0$. With biased diffusion, we find the mean-field exponents with logarithmic corrections for any finite r, but the DP is robust against this external driving at $r = 0$ [23]. This again confirms that the PCPD (in general, GPCPD at nonzero r) should not belong to the DP class.

In conclusion, we studied the effect of bias on the critical scaling in one-dimensional reaction-diffusion models. The BAW models are robust against the external driving, regardless of the parity conservation. This is anticipated from the fact that the DP and the PC class can be generically described by a single-species bosonic field theory, where the Galilean invariance is embedded. In contrast, the driving is relevant for the PCPD and changes the critical scaling. This leads us to exclude a possibility of the DP or the PC class for the critical scaling of the PCPD model. Moreover, it suggests that the PCPD is generically a two-species model and a field theory of two-species may be required.

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